Student Workbook

Rates of Change

Name: __________

Date: __________

Period: __________

Learning Target: I can find the average rate of change of a function over an interval and identify where a function is increasing or decreasing.

Vocabulary

Term	Meaning
Average Rate of Change	How much a function's output changes, on average, per unit of input over an interval.
Secant Line	A line connecting two points on a curve. Its slope equals the average rate of change between those points.
Increasing/Decreasing Interval	An interval of x-values where the function's y-values go up (increasing) or down (decreasing) as x increases.
Difference Quotient	$[f(x+h) - f(x)] / h$ — the average rate of change formula written generally, a building block for calculus.

Key Concept: Average Rate of Change

FORMULA

Average rate of change from $x = a$ to $x = b$: **$(f(b) - f(a)) / (b - a)$**

This is exactly the slope formula applied to two points on the function's graph — it's the slope of the secant line through those points.

EXAMPLE 1

Find the average rate of change of $f(x) = x^2 - 2x$ from $x = 1$ to $x = 4$.

$f(1)$ = ______ $f(4)$ = ______

Rate = $(f(4) - f(1)) / (4 - 1)$ = ____________

COMMON MISTAKE

Keep your order consistent! If you compute $f(b) - f(a)$ on top, you MUST compute $b - a$ (not $a - b$) on the bottom, or your sign will flip.

Key Concept: Increasing & Decreasing Intervals

READING A GRAPH

Scan the graph from LEFT to RIGHT. If the y-values are going UP, the function is increasing on that interval. If they're going DOWN, it's decreasing.

EXAMPLE 2

A graph shows a curve increasing from $x = -\infty$ to $x = -2$, then decreasing from $x = -2$ to $x = 3$, then increasing again from $x = 3$ to $x = \infty$.

Increasing: ____________________

Decreasing: ____________________

Local maximum at x = _______ Local minimum at x = _______

Key Concept: The Difference Quotient

FORMULA

$[f(x+h) - f(x)] / h$ is the average rate of change over the interval from x to x+h. As h gets smaller and smaller, this approaches the **instantaneous** rate of change — the foundation of calculus!

EXAMPLE 3

Find the difference quotient for $f(x) = x^2 + 4x$.

$f(x+h) = (x+h)^2 + 4(x+h) = x^2 + 2xh + h^2 + 4x + 4h$

$f(x+h) - f(x) = 2xh + h^2 + 4h$

$[f(x+h) - f(x)] / h =$ ____________________

Guided Practice — Part A: Average Rate of Change from a Function

A1 $f(x) = x^2 + 1$, from $x = 0$ to $x = 3$

A2 $f(x) = 2x^2 - x$, from $x = -1$ to $x = 2$

Guided Practice — Part B: Average Rate of Change from a Table

B1 Find the average rate of change from $x=2$ to $x=6$.

x	0	2	4	6
y	5	9	15	23

Guided Practice — Part C: Increasing/Decreasing Intervals

C1 A graph increases on $(-\infty,-2)$, decreases on $(-2,3)$, and increases on $(3,\infty)$. Where does a local max occur? A local min?

Guided Practice — Part D: The Difference Quotient

D1 Find the difference quotient for $f(x) = x^2 + 4x$.

D2 Find the difference quotient for $f(x) = 3x + 5$.

This is a linear function — what do you notice about your answer?

BEFORE YOU MOVE ON

Check with a partner: for a straight line, the difference quotient should simplify to a constant. Did yours?